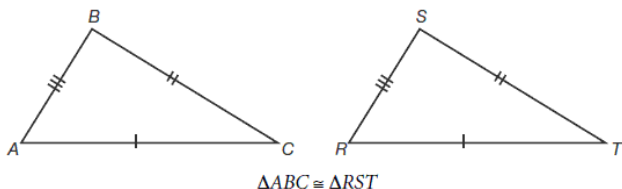


Triangle Congruence Theorems

Draw an example of each triangle congruency. The first one has been done for you.

1) Two triangles congruent by SSS.



2) Two triangles congruent by SAS.

3) Two triangles congruent by AAS.

4) Two triangles congruent by ASA.

5) Two triangles are congruent by HL.

6) Which of the following is NOT a triangle congruence theorem?

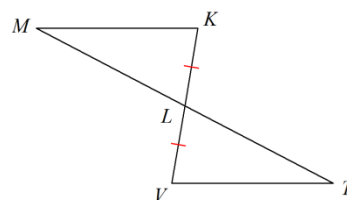
- a. SSA
- b. AAS
- c. SSA
- d. ASA

Quick example proof! **READ CAREFULLY TO RE-TEACH YOURSELF STEPS.**

Given $\angle K \cong \angle V$

Prove $\Delta MKL \cong \Delta TVL$

Statement	Reason
1) $\angle K \cong \angle V$	1) Given
2) $KL \cong LV$	2) Given within figure
3) $\angle KLM \cong \angle VLT$	3) Vertical angles
4) $\Delta MKL \cong \Delta TVL$	4) ASA

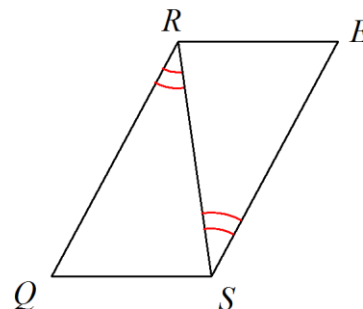


Now you!

7) **Given** $QR \cong ES$

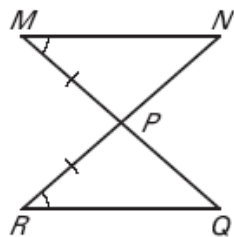
Prove $\Delta QRS \cong \Delta ESR$

Statement	Reason
1)	1)
2)	2)
3)	3)
4)	4)



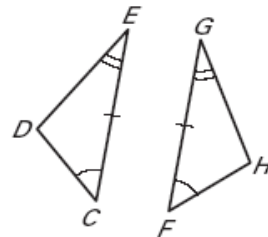
8) In the figure below, which of the following congruencies must be true?

- a) $MP \cong PQ$
- b) $RP \cong PQ$
- c) $MP \cong NP$
- d) $MN \cong RQ$
- e) $RP \cong PN$



9) Which of the following is true according to CPCTC?

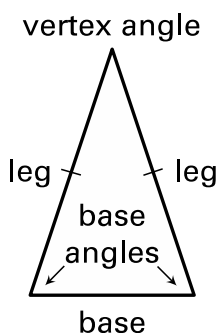
- a) $DE \cong DC$
- b) $GF \cong DE$
- c) $HF \cong HF$
- d) $DC \cong GH$
- e) $ED \cong GH$



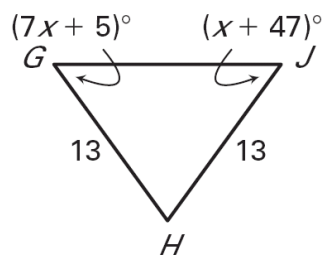
Isosceles Triangles

10) Fill in the blanks below.

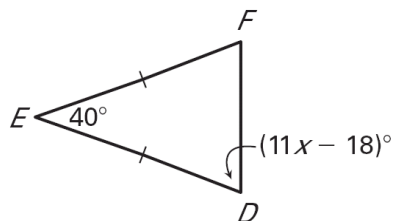
An isosceles triangle has two congruent _____ angles and one _____ angle. It also has two congruent sides called _____.



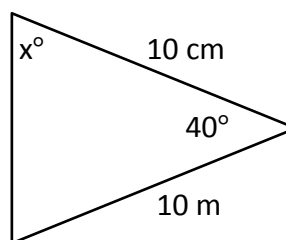
11) Find the value of x.



12) Find the value of x.



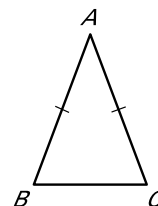
13) In the triangle shown below, what is the value of x?



THEOREM 4.7: BASE ANGLES THEOREM

If two sides of a triangle are congruent, then the angles opposite them are congruent.

If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong$ ____.

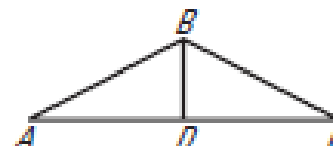


Quick example proof! **READ CAREFULLY TO RE-TEACH YOURSELF STEPS.**

GIVEN: $AB \cong BC$

PROVE: $\angle A \cong \angle C$

Statement	Reason
1) $AB \cong BC$	1) Given
2) $\angle A \cong \angle C$	2) Base Angles Theorem



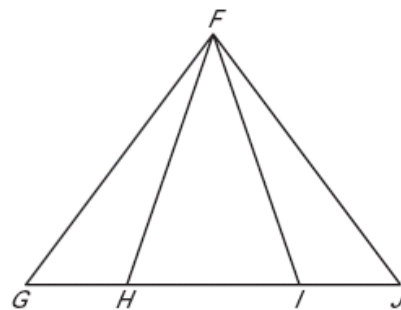
Now you!

14)

GIVEN: $\overline{FG} \cong \overline{FJ}$, $\overline{HG} \cong \overline{IJ}$

PROVE: $\overline{HF} \cong \overline{IF}$

Statement	Reason
1)	1)
2)	2)
3)	3)
4)	4)



Proportions & Similar Triangles

Example:

You are painting barn doors. You know that the perimeter of the doors is 64 feet and that the ratio of the length to the width is 3:5. Find the area of the doors.

$$P = 2L + 2W$$

$$64 = 2(3x) + 2(5x)$$

$$64 = 6x + 10x$$

$$64 = 16x$$

$$4 = x$$

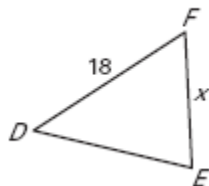
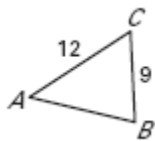
$$\text{Length: } 5(4) = 20 \text{ ft} \quad \text{Width: } 3(4) = 12 \text{ ft}$$

$$\text{Area: } (20 \text{ ft})(12 \text{ ft}) = \boxed{240 \text{ ft}^2}$$

Now you!

15) The perimeter of a rectangular table is 21 feet and the ratio of its length to its width is 5:2. Find the area of the table.

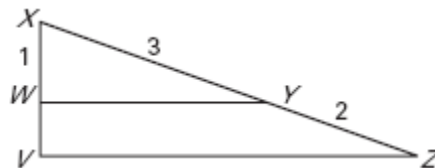
16) Find x.



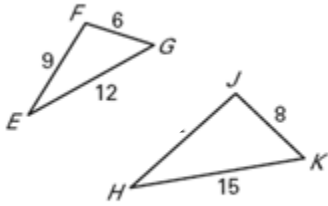
$$\frac{AC}{DF} = \frac{BC}{EF}$$

17) Find WV.

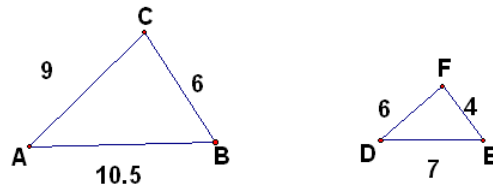
$$\frac{XW}{WV} = \frac{XY}{YZ}$$



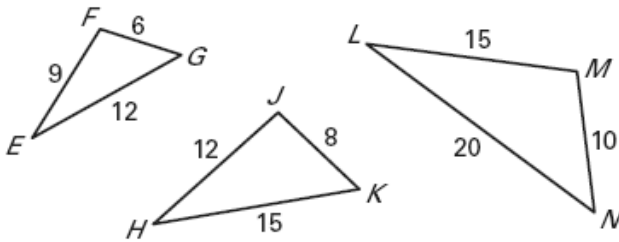
18) Triangles FEG and JHK are similar. Find the length of JH.



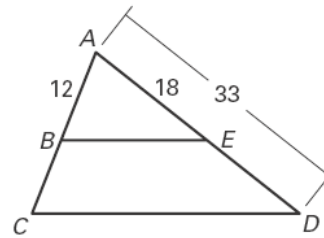
19) Check to see if the following two triangles are similar. If so, write a similarity statement.



20) Which of the following triangles are similar?



21) BE is parallel to CD. Find the measure of BC.

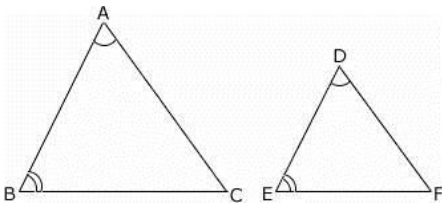


Triangle Similarity Theorems

Theorems SSS, SAS, ASA, AAS, and HL prove that two triangles are **congruent**, but you can also prove that two triangles are **similar** by AA, SSS, and SAS.

Draw an example of each triangle similarity. The first one has been done for you.

22) Two triangles similar by AA.



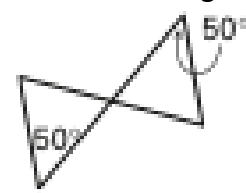
23) Two triangles similar by SSS.

23) By what similarity theorem are these two triangles similar?

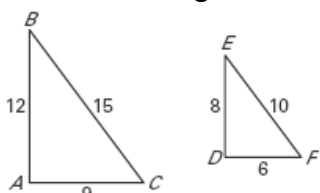


24) Two triangles similar by SAS.

24) By what similarity theorem are these two triangles similar?



25) By what similarity theorem are these two triangles similar?



45-45-90 Triangles

25) Fill in the blanks below.

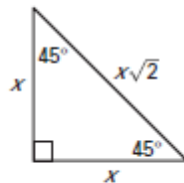
Your Notes

The extended ratio of the side lengths of a 45°-45°-90° triangle is $1:1:\sqrt{2}$.

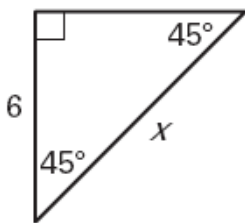
THEOREM 7.8: 45°-45°-90° TRIANGLE THEOREM

In a 45°-45°-90° triangle, the hypotenuse is _____ times as long as each leg.

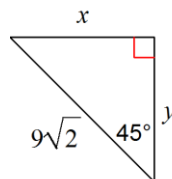
hypotenuse = leg • _____



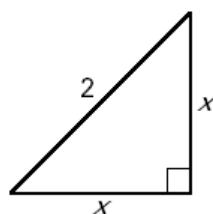
26) Find the value of x . Leave your answer in simplest radical form.



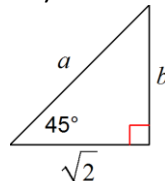
27) Find the value of x and y .



28) Find the value of x . Leave your answer in simplest radical form.



29) Find the value of a and b .



30-60-90 Triangles

30) Fill in the blanks below.

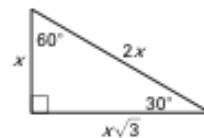
The extended ratio of the side lengths of a 30°-60°-90° triangle is $1:\sqrt{3}:2$.

THEOREM 7.9: 30°-60°-90° TRIANGLE THEOREM

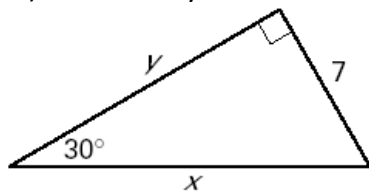
In a 30°-60°-90° triangle, the hypotenuse is _____ as long as the shorter leg, and the longer leg is _____ times as long as the shorter leg.

hypotenuse = _____ • shorter leg

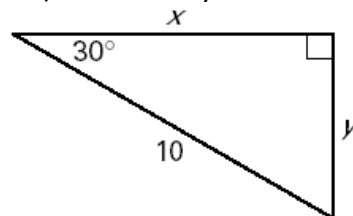
longer leg = shorter leg • _____



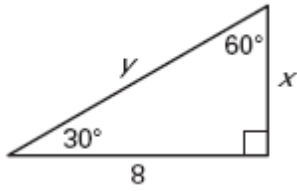
31) Find x and y .



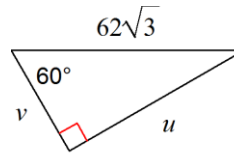
32) Find x and y .



33) Find x and y . Leave your answer in simplest radical form.



34) Find u and v . Leave your answer in simplest radical form.



SOHCAHTOA

35) Write out what SOHCAHTOA stands for on the lines below.

S _____
 O _____
 H _____
 C _____
 A _____
 H _____
 T _____
 O _____
 A _____

36) Find each trigonometric ratio for the given right triangle.

$$\sin Z = \frac{8}{17}$$

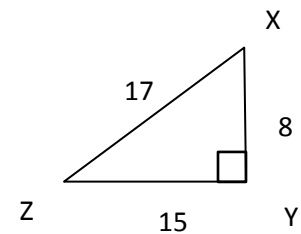
$$\cos Z = \underline{\hspace{2cm}}$$

$$\tan Z = \underline{\hspace{2cm}}$$

$$\sin X = \underline{\hspace{2cm}}$$

$$\cos X = \underline{\hspace{2cm}}$$

$$\tan X = \underline{\hspace{2cm}}$$



37) An angle in a right triangle has a measure θ .
 If $\sin \theta = \frac{12}{13}$, then $\tan \theta = ?$

(Hint: draw a diagram of what you are given first. See if you can figure out the rest of the information about the triangle to find $\tan \theta$.)

38) In the figure below, $\triangle ABC$ is a right triangle with a right angle at C . Which of the statements about this figure is NOT correct?

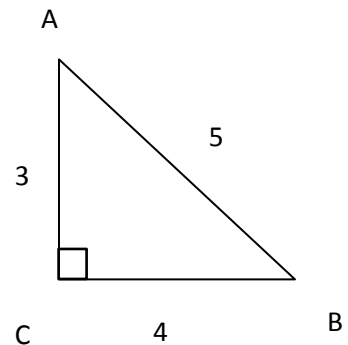
a. $\cos A = \frac{3}{5}$

b. $\sin A = \frac{4}{5}$

c. $\tan A = \frac{3}{4}$

d. $\cos B = \frac{4}{5}$

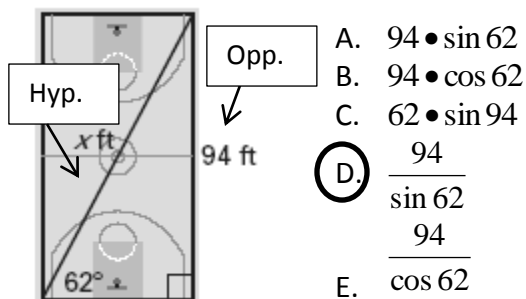
e. $\tan B = \frac{3}{4}$



Finding Missing Sides Using Trig

Example:

You walk from one corner of a basketball court to the opposite corner. Which of the following would solve for the distance x ?



$$\sin = \frac{\text{opp}}{\text{hyp}}$$

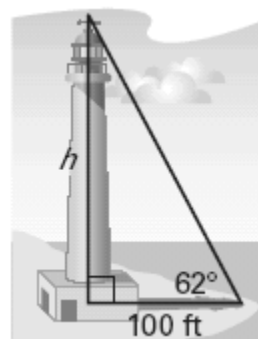
$$\sin 65 = \frac{94}{x}$$

$$x \sin 65 = 94$$

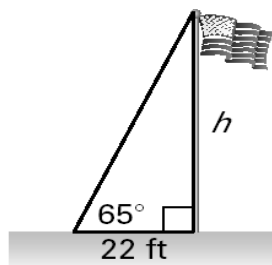
$$x = \frac{94}{\sin 65}$$

39) Which of the following would find the height of the lighthouse?

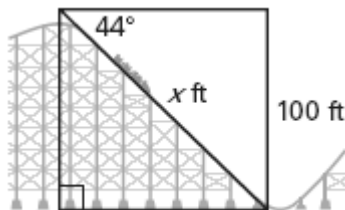
- A. $62 \cdot \tan 100$
- B. $100 \cdot \tan 62$
- C. $100 \cdot \sin 62$
- D. $\frac{100}{\sin 62}$
- E. $\frac{100}{\tan 62}$



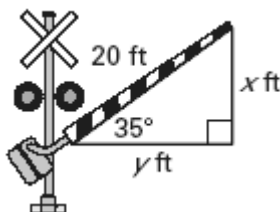
40) To calculate the height h of a flagpole, you move 22 feet from the base and record the angle of elevation to the top to be 65° . Find the flagpole's height to the nearest foot. (Use your calculator.)



41) You are at the top of a roller coaster 100 feet above the ground. The angle of depression is 44° . What is the distance x to the nearest foot?



42) A railroad crossing arm that is 20 feet long is stuck with an angle of elevation of 35° . Find the lengths x and y to the nearest foot.



Finding Missing Angles Using Trig

To solve for a degree measure like θ , we will have to use: **inverse** sine/cosine/tangent.

To do this, you must use the **inverse** function on your calculator.

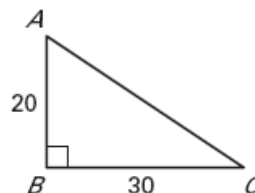
Practice:

$$\tan \theta = \frac{3}{4}$$

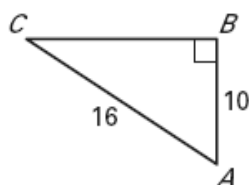
$$\theta = \tan^{-1} \frac{3}{4}$$

$$\theta = \underline{\hspace{2cm}}^\circ$$

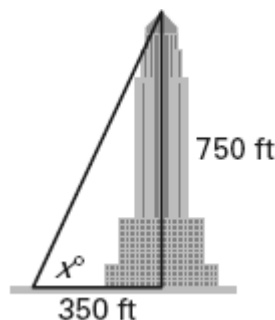
44) Use a calculator to approximate the measure of angle A to the nearest degree.



45) Use a calculator to approximate the measure of angle A to the nearest degree.



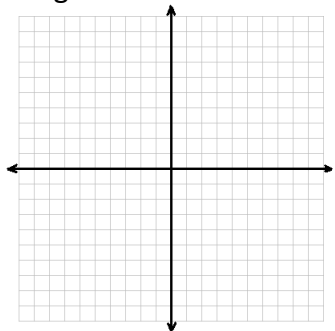
46) You are standing 350 feet away from a skyscraper that is 750 feet tall. What is the angle of elevation from you to the top of the building?



47) Fill in the blanks below with either “inverse” or “regular.”

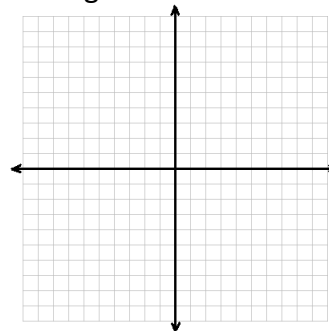
- When solving for a side length, use _____ trig function.
- When solving for an angle, use _____ trig function.

48) Draw a 50° angle.

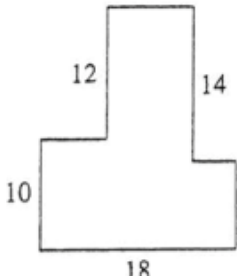
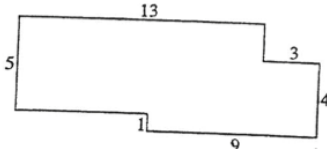


a. Find **two** positive angles that are coterminal with the given angle.

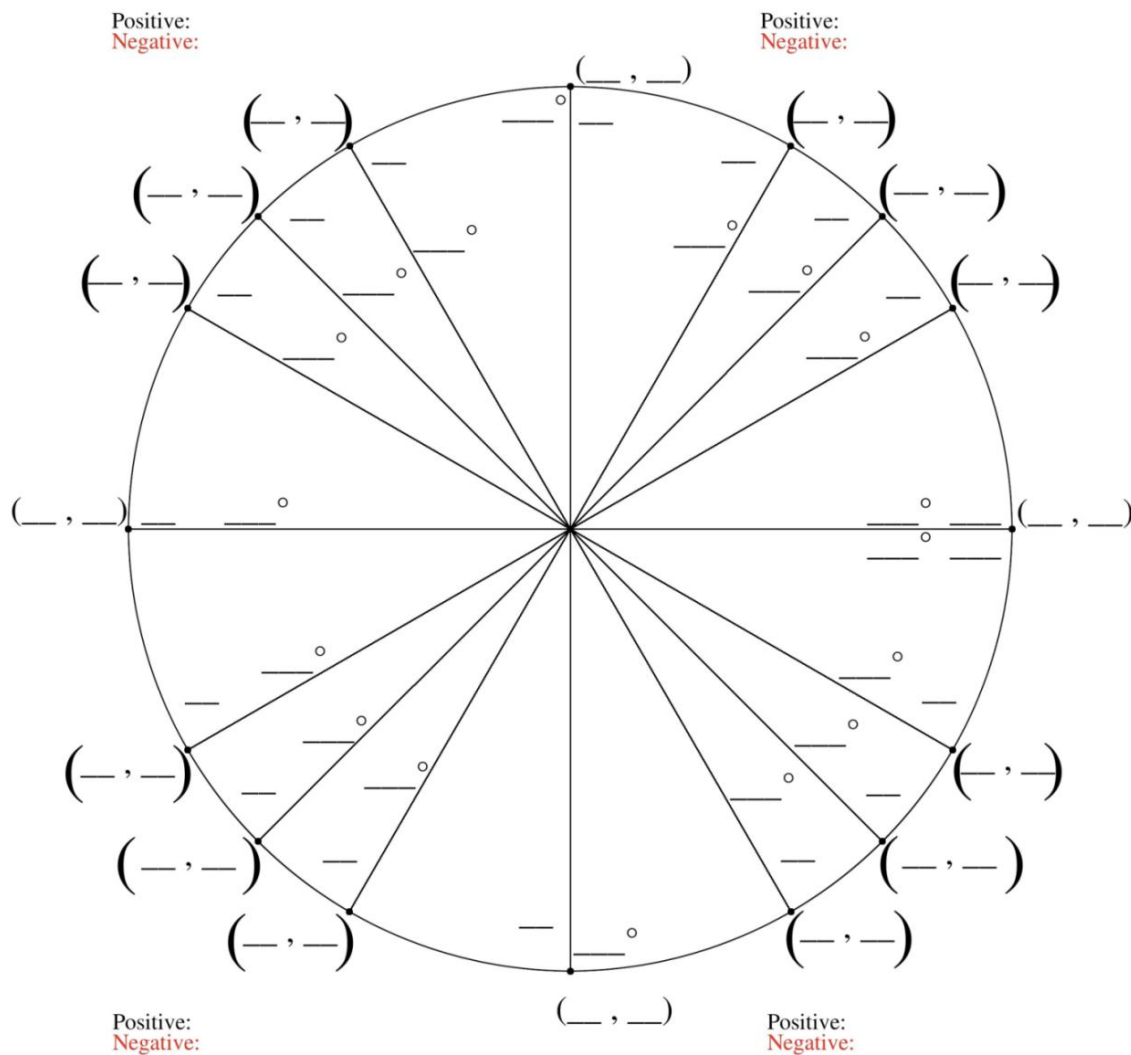
49) Draw a -230° angle.



a. Find **two** positive angles that are coterminal with the given angle.

b. Find two negative angles that are coterminal with the given angle.	b. Find two negative angles that are coterminal with the given angle.
50) Convert 240° to radians.	51) Convert 90° to radians.
52) Convert 30° to radians.	53) Convert 135° to radians.
54) Convert $\frac{\pi}{2}$ radians to degrees.	55) Convert $\frac{3\pi}{2}$ radians to degrees.
56) Convert $\frac{3\pi}{2}$ radians to degrees.	57) Convert $\frac{5\pi}{6}$ radians to degrees.
<p>58)</p> <p>In the 8-sided figure below, adjacent sides meet at right angles and the lengths given are in meters. What is the perimeter of the figure, in meters?</p> 	<p>59) Find the perimeter of the figure below.</p> 

60) Fill in the unit circle.



61) A park wants to put a fence around a baseball field in the shape below. Each straight side of the fence is 120 feet long, and the rounded side is an arc that measures 90° . How many feet of fencing does the park need to purchase?

Exact _____
Approximate _____

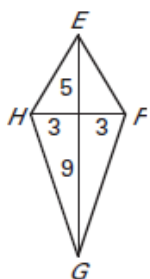
62) The city of Mount Prospect is trying to build a circular field. They want to build a fence around the entire field. How much of fencing is needed if the field if the diameter was 18 meter?

Exact _____
Approximate _____

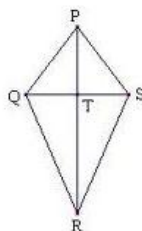
63) The perimeter of a rectangle is 50 inches. The width is unknown. The length is two times longer than the width. Find the area of the rectangle.

64) The perimeter of a rectangle is 64 feet. The width of the rectangle is half as long as its length. Find the length and width of the rectangle.

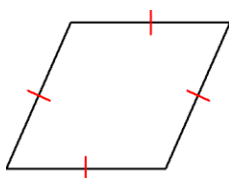
65) Find the area of the kite below.



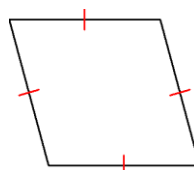
66) In the kite below, $PQ = 5$ cm, $PS = 5$ cm, $QS = 6$ cm, and $TR = 12$ cm. What is the area of the kite?



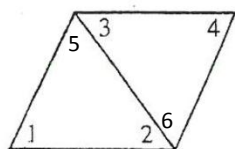
67) A rhombus has diagonals of 20 ft and 34 ft. What is the area of the rhombus?



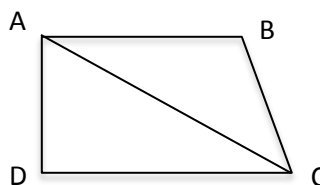
68) A rhombus has four equal side lengths of 13 in. The length of the longer diagonal is 24 in. What is the area of the rhombus?



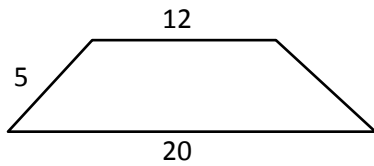
69) In the parallelogram below, a diagonal is shown and $\angle 2$ measures 63° and $\angle 6$ measures 42° . What is the $m\angle 4$?



70) In the trapezoid ABCD shown below, side lengths AB and DC are parallel, $\angle DAC = 72^\circ$, $\angle D = 90^\circ$, and $\angle B = 109^\circ$. What is the measure of $\angle BCA$?

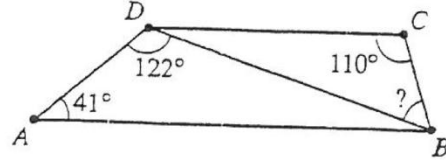


71) The parallel sides of the isosceles trapezoid shown below are 12 feet long and 20 feet long, respectively. What is the distance, in feet, between these two sides?



72)

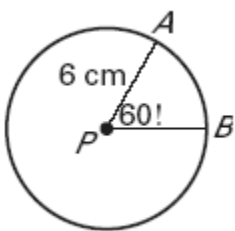
In the figure below, $\overline{AB} \parallel \overline{DC}$, $\angle A$ measures 41° , $\angle C$ measures 110° , and $\angle ADB$ measures 122° . What is the measure of $\angle CBD$?



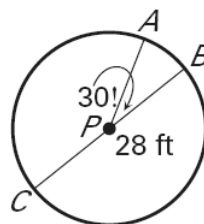
73) Fill in the chart according to the properties of the quadrilaterals:

	square	rhombus	parallelogram	rectangle	kite	trapezoid	isosceles trap.
diagram							
# of parallel sides							
# of congruent sides							
diagonals \perp ?							
diagonals \cong ?							

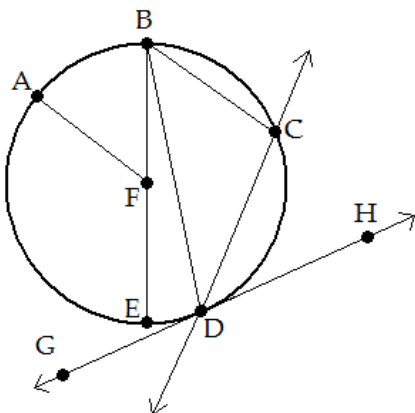
74) Find the length of arc AB.



75) Find the length of arc AB.

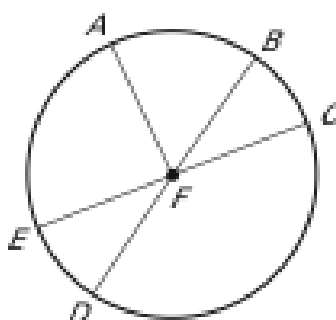


76) Use the figure below to answer the following questions.



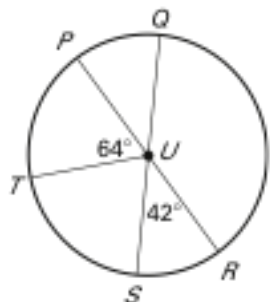
- a. \widehat{AEC} is a _____.
- b. \widehat{BD} is a _____.
- c. \widehat{EB} is a _____.

77) Determine whether the arc is a major arc, minor arc, or semicircle.



- a. arc EDC
- b. arc EB
- c. arc BD
- d. arc AC

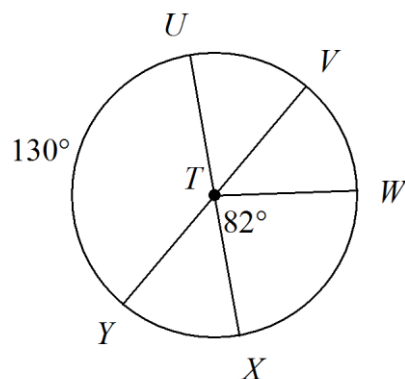
78) In the figure, PR and QS are diameters of circle U. Find the measure of the indicated arc.



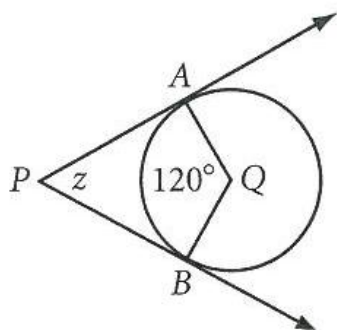
- | | |
|----------------------|----------------------|
| 9. $m\widehat{PQ}$ | 10. $m\widehat{ST}$ |
| 11. $m\widehat{TPS}$ | 12. $m\widehat{RT}$ |
| 13. $m\widehat{RQS}$ | 14. $m\widehat{QR}$ |
| 15. $m\widehat{PQS}$ | 16. $m\widehat{TOR}$ |
| 17. $m\widehat{PS}$ | 18. $m\widehat{PTR}$ |

79. Find the measure of the arc or central angle indicated. Assume that lines which appear to be diameters are actual diameters.

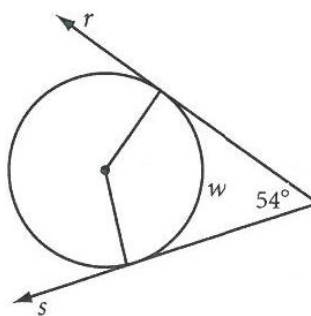
$m\angle UTV$



80) PA and PB are tangent to circle Q. Find z.



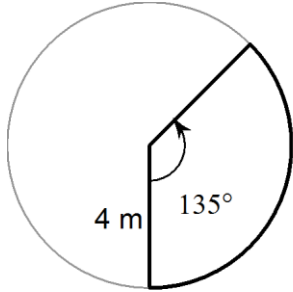
81) Lines r and s are tangent to the circle. Find w.



82) A pizza is sitting in a circular box that fits its edges exactly. The pizza has a diameter that is 12 inches across. The pizza has 8 equal slices cut at equal angles through the center of the pizza. When you take one piece of pizza from the circular pizza box, what is the approximate length of the exposed side of the box?

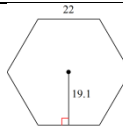
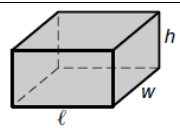
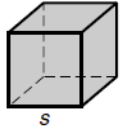
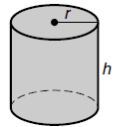
83) The minute hand and the hour hands on a clock originate at the center of the clock and reach to the outer edge of the clock. When it is 2:00 pm, what is the length of the arc created by the minute hand and the hour hand?

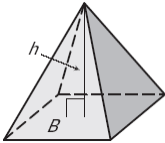
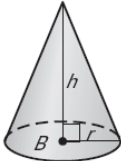

84) Find the area of the sector.



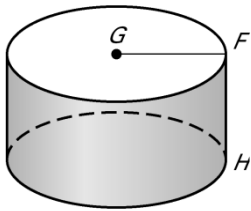
85) A child's bicycle wheel completes exactly three revolutions when it travels 150 cm. The wheel has a decoration on it that covers exactly half of the inner part of the wheel. What is the area of the decoration? Leave your answer in terms of pi.

Surface Area & Volume Formulas

Solid	Formula: SURFACE AREA	Formula: VOLUME	Picture
REGULAR POLYGON			
RECTANGULAR PRISM			
CUBE			
CYLINDER			

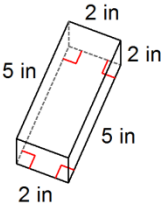
PYRAMID			
CONE			
SPHERE			

86) Use the diagram at the below to answer the questions at the right.



- Give the mathematical name of the solid.
- What kind of figure is each base?
- Name the radius of the solid.
- Name the height of the solid.

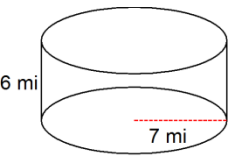
87) Find the surface area and volume of the rectangular prism below.



Surface Area:

Volume:

88) Find the surface area and volume of the cylinder below and leave it in terms of π .



Surface Area:

Volume:

89) A cube has a volume of 1728 cm^3 .

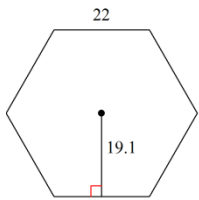
a. What is the area of one face of the cube?

b. What is the full surface area of the cube?

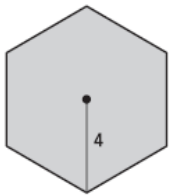
90) The surface area of one face of a cube is 121 m^2 . Find the volume of the cube.

91) Find the area of the regular polygons below. All units are in inches.

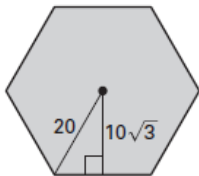
a.



b. Find the area of the regular polygon below. Each side is 5 cm. Round to the nearest hundredth.



c.



92)

a. In #1 problem (a) I was given (circle 2):

Apothem "Radius" Side length

So I solved by _____

_____.

b. In #1 problem (b) I was given (circle 2):

Apothem "Radius" Side length

So I solved by _____

_____.

Continued →

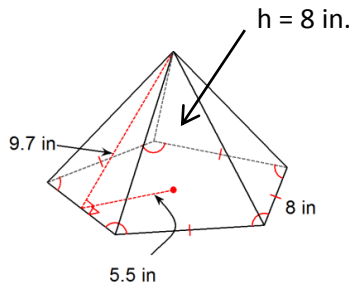
a. In #1 problem (c) I was given (circle 2):

Apothem "Radius" Side length

So I solved by _____

_____.

93) Find the surface area and volume of the pyramid below.



Surface area: _____ Volume: _____

94)

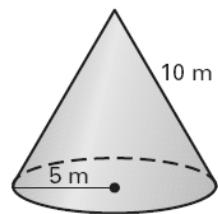
- Which of the values shown in # 3 is the **lateral** height of the pyramid?

- Which of the following formulas requires **lateral** height for a pyramid—surface area or volume?

- Which of the values shown in # 3 is the **actual** height of the pyramid?

- Which of the following formulas requires **actual** height for a pyramid—surface area or volume?

95) Find the surface area and volume of the cone below.



Surface area: _____ Volume: _____

96)

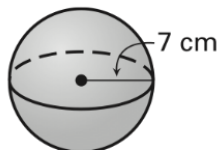
- Which of the values shown in # 5 is the **lateral** height of the cone?

- Which of the following formulas requires **lateral** height for a cone—surface area or volume?

- Which of the values shown in # 5 is the **actual** height of the cone?

- Which of the following formulas requires **actual** height for a cone—surface area or volume?

97) Find the surface area and the volume of the sphere below.



Surface area: _____ Volume: _____

98) State the error based on # 7.

$$V = \frac{4}{3} \pi (7)^2$$

$$V = \frac{4}{3} \pi (49)$$

$$V = 579.1 \text{ in}^3$$

Error: _____

<p>99) The length of each side of a cube is increased by 4 times. By how many times did the volume increase?</p>	<p>100) A cylinder has a radius of 4 ft and height of 10 ft. If the radius is increased by 2 times, find the ratio of the old volume to the new volume.</p>
<p>101) The length of each side of a cube is multiplied by 3. What is the change in <i>volume</i> of the cube?</p> <p>A. The volume is 3 times greater. B. The volume is 6 times greater. C. The volume is 9 times greater. D. The volume is 27 times greater.</p>	<p>102) A cylinder has a height of 4 cm and a radius of 2 cm. If both the height and radius are doubled, what is the ratio of the volume of the original cylinder to the volume of the new cylinder?</p>